APPENDIX A: THE SURFACES

In this section we describe all the surfaces listed in Table 1, 2 and 3

K_S^2	Sing(X)	Sign.	G^0	G	$H_1(S,\mathbb{Z})$	$\pi_1(S)$	Label
1	$2\,C_{2,1}, 2\,D_{2,1}$	$2^3, 4$	$D_4 \times \mathbb{Z}_2$	$\mathbb{Z}_2^3 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	1.1
2	$6\,C_{2,1}$	2^5	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	1.2
2	$6\,C_{2,1}$	4^3	$(\mathbb{Z}_2 \times \mathbb{Z}_4) \rtimes \mathbb{Z}_4$	G(64,82)	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$	1.3
2	$C_{2,1}, 2\,D_{2,1}$	$2^3, 4$	$\mathbb{Z}_2^4 \rtimes \mathbb{Z}_2$	$\mathbb{Z}_2^4 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	1.4
2	$C_{2,1}, 2\,D_{2,1}$	$2^2, 3^2$	$\mathbb{Z}_3^2 \rtimes \mathbb{Z}_2$	$\mathbb{Z}_3^2 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_3$	$\mathbb{Z}_3$	1.5
2	$2\,C_{4,1}, 3\,C_{2,1}$	$2^3, 4$	G(64,73)	G(128,1535)	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$	1.6
2	$2\,C_{3,1}, 2\,C_{3,2}$	$3^2, 4$	G(384,4)	G(768,1083540)	$\mathbb{Z}_4$	$\mathbb{Z}_4$	1.7
2	$2\,C_{3,1}, 2\,C_{3,2}$	$3^2, 4$	G(384,4)	G(768,1083541)	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	1.8
3	$C_{8,3}, C_{8,5}$	$2^3, 8$	G(32, 39)	G(64, 42)	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	1.9
4	$4\,C_{2,1}$	2^5	$D_4 \times \mathbb{Z}_2$	$D_{2,8,5} \rtimes \mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_8$	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_8$	1.10
4	$4\,C_{2,1}$	2^5	$\mathbb{Z}_2^4$	$(\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4) \times \mathbb{Z}_2$	$\mathbb{Z}_2^3 \times \mathbb{Z}_4$	∞	1.11
4	$4\,C_{2,1}$	4^3	G(64, 23)	G(128, 836)	$\mathbb{Z}_2^3$	$\mathbb{Z}_4^2 \rtimes \mathbb{Z}_2$	1.12
8	$\emptyset$	2^5	$D_4 \times \mathbb{Z}_2^2$	$(D_{2,8,5} \rtimes \mathbb{Z}_2) \times \mathbb{Z}_2$	$\mathbb{Z}_2^3 \times \mathbb{Z}_8$	∞	1.13
8	$\emptyset$	4^3	G(128, 36)	G(256, 3678)	$\mathbb{Z}_4^3$	∞	1.14
8	$\emptyset$	4^3	G(128, 36)	G(256, 3678)	$\mathbb{Z}_2^4 \times \mathbb{Z}_4$	∞	1.15
8	$\emptyset$	4^3	G(128, 36)	G(256, 3678)	$\mathbb{Z}_2^2 \times \mathbb{Z}_4^2$	∞	1.16
8	$\emptyset$	4^3	G(128, 36)	G(256, 3679)	$\mathbb{Z}_2^2 \times \mathbb{Z}_4^2$	∞	1.17

TABLE 1. $p_g = q = 0$

K_S^2	g_{alb}	Sing(X)	Sign.	G^0	G	$H_1(S,\mathbb{Z})$	Label
2	2	$C_{2,1}, 2D_{2,1}$	2^2	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}^2$	2.1
2	2	$C_{2,1}, 2D_{2,1}$	2	D_8	$D_{2,8,3}$	$\mathbb{Z}^2$	2.2
2	2	$C_{2,1}, 2D_{2,1}$	2	Q_8	BD_4	$\mathbb{Z}^2$	2.3
4	3	$4C_{2,1}$	2^2	$\mathbb{Z}_4$	$\mathbb{Z}_8$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.4
4	3	$4C_{2,1}$	2^2	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.5
4	2	$4C_{2,1}$	2	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4$	G(32,29)	$\mathbb{Z}_2^2 \times \mathbb{Z}^2$	2.6
4	3	$4C_{2,1}$	2	$D_{4,4,3}$	$D_{4,8,3}$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.7
4	3	$4C_{2,1}$	2	$D_{4,4,3}$	$D_{4,8,7}$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.8
4	2	$4C_{2,1}$	2	$D_{4,4,3}$	G(32,32)	$\mathbb{Z}_2^2 \times \mathbb{Z}^2$	2.9
4	2	$4C_{2,1}$	2	$D_{4,4,3}$	G(32,35)	$\mathbb{Z}_2^2 \times \mathbb{Z}^2$	2.10
4	3	$4C_{2,1}$	2	$D_{2,8,5}$	G(32,15)	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.11
5	3	$C_{3,1}, C_{3,2}$	3	BD_3	BD_6	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.12
5	3	$C_{3,1}, C_{3,2}$	3	D_6	$D_{2,12,5}$	$\mathbb{Z}_2^2 \times \mathbb{Z}^2$	2.13
6	3	$2C_{2,1}$	2	$A_4 \times \mathbb{Z}_2$	$A_4 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.14
6	7	$2C_{2,1}$	2	$A_4 \times \mathbb{Z}_2$	$A_4 \times \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.15
6	5	$C_{5,3}$	5	D_5	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^2$	2.16
8	5	$\emptyset$	2^2	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$D_{2,8,5}$	$\mathbb{Z}_4 \times \mathbb{Z}^2$	2.17
8	5	$\emptyset$	2^2	D_4	$D_{2,8,3}$	$\mathbb{Z}_4 \times \mathbb{Z}^2$	2.18
8	5	$\emptyset$	2^2	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4$	$\mathbb{Z}_2^3 \times \mathbb{Z}^2$	2.19

TABLE 2. $p_g = q = 1$

K_S^2	Sing(X)	Sign.	G^0	G	$H_1(S,\mathbb{Z})$	Label
8	$\emptyset$	-	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_2 \times \mathbb{Z}^4$	3.1

TABLE 3. $p_g = q = 2$

We will follow the scheme below:
G: the Galois group.
G^0: the index 2 subgroup of the elements that do not exchanges the factors.
T: the type of the generating vector $(m_1, \dots, m_r)$.
L: here we list the set of elements of G that is a generating vector for G^0 that gives the curve C: $(a_1, b_1, \dots, a_q, b_q, c_1, \dots, c_r)$.
H_1: the first homology group of the surface.
π_1: the fundamental group of the surface (only if $q \neq 0$).

1. $p_g = q = 0$

$K^2 = 1$, Basket $\{2 \times C_{2,1},\ 2 \times D_{2,1}\}$.

1.1. Galois group G(32,6): $(\mathbb{Z}_2)^3 \rtimes_\varphi \mathbb{Z}_4 \,:\, \varphi(1) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

G: $\langle (2,5,6,8)(3,7),\ (1,2)(3,5)(4,6)(7,8),\ (1,3)(2,5)(4,7)(6,8),$
$(2,6)(5,8),\ (1,4)(2,6)(3,7)(5,8) \rangle < \mathfrak{S}_8$
G^0: *G(16,11)*: $D_4 \times \mathbb{Z}_2$
T: $(2,2,2,4)$

L: $(1,8)(2,7)(3,6)(4,5)$, $(1,7)(2,8)(3,4)(5,6)$, $(1,3)(2,8)(4,7)(5,6)$, $(1,5,4,8)(2,7,6,3)$
H_1: $\mathbb{Z}_4$
π_1: $\mathbb{Z}_4$

$K^2 = 2$, Basket $\{6 \times C_{2,1}\}$.

1.2. Galois group G(16,3): $(\mathbb{Z}_2)^2 \rtimes_\varphi \mathbb{Z}_4$: $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

G: $\langle (1,2,4,6)(3,5,7,8), (2,5)(6,8), (1,3)(2,5)(4,7)(6,8), (1,4)(2,6)(3,7)(5,8) \rangle < \mathfrak{S}_8$
G^0: *G(8,5)*: $(\mathbb{Z}_2)^3$
T: $(2,2,2,2,2)$
L: $(1,3)(4,7)$, $(1,7)(2,6)(3,4)(5,8)$, $(1,3)(2,5)(4,7)(6,8)$, $(2,5)(6,8)$, $(1,7)(2,6)(3,4)(5,8)$
H_1: $\mathbb{Z}_2 \times \mathbb{Z}_4$
π_1: $\mathbb{Z}_2 \times \mathbb{Z}_4$

1.3. Galois group G(64,82): Sylow 2-subgroup of the Suzuki group $Sz(8)$,

G: $\langle g_1, g_2, g_3 \mid g_3^4, g_2^4, g_1^4, g_1 g_3 g_1^{-1} g_3 g_2^2, g_2^{-2} g_3^{-1} g_1^{-1} g_3^{-1} g_1, g_2 g_3 g_1^2 g_2 g_3^{-1}, g_1^{-1} g_3^2 g_2 g_1 g_2^{-1}, g_2^{-1} g_3^2 g_2 g_3^2, g_1^{-2} g_3^{-1} g_2 g_3 g_2 \rangle$
G^0: *G(32,2)*: $(\mathbb{Z}_2 \times \mathbb{Z}_4) \rtimes_\varphi \mathbb{Z}_4$ where $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$
T: $(4,4,4)$
L: g_3^{-1}, $g_1 g_3^{-2}$, $g_1 g_3 g_2^{-2} g_3^2 g_2^2 g_1^{-2}$
H_1: $(\mathbb{Z}_2)^3$
π_1: $(\mathbb{Z}_2)^3$

$K^2 = 2$, Basket $\{C_{2,1},\, 2 \times D_{2,1}\}$.

1.4. Galois group G(64,32): $(\mathbb{Z}_2)^4 \rtimes_\varphi \mathbb{Z}_4$: $\varphi(1) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

G: $\langle (2,6,7,12)(3,9,10,16)(4,11)(8,14,15,13)$,
$(1,2)(3,6)(4,7)(5,8)(9,13)(10,14)(11,15)(12,16)$,
$(1,3)(2,6)(4,9)(5,10)(7,13)(8,14)(11,16)(12,15)$,
$(2,7)(3,10)(6,12)(8,15)(9,16)(13,14)$,
$(1,4)(2,7)(3,9)(5,11)(6,13)(8,15)(10,16)(12,14)$,
$(1,5)(2,8)(3,10)(4,11)(6,14)(7,15)(9,16)(12,13) \rangle < \mathfrak{S}_{16}$
G^0: *G(32,27)*: $(\mathbb{Z}_2)^4 \rtimes_\psi \mathbb{Z}_2$, $\psi(1) = \varphi(2)$
T: $(2,2,2,4)$
L: $(2,7)(3,10)(6,12)(8,15)(9,16)(13,14)$,
$(1,16)(2,12)(3,11)(4,10)(5,9)(6,15)(7,14)(8,13)$,
$(1,14)(2,10)(3,8)(4,12)(5,6)(7,16)(9,15)(11,13)$,

$(1,2,4,7)(3,14,9,12)(5,8,11,15)(6,16,13,10)$
H_1: $\mathbb{Z}_4$
π_1: $\mathbb{Z}_4$

1.5. Galois group G(36,9): $(\mathbb{Z}_3)^2 \rtimes_\varphi \mathbb{Z}_4$: $\varphi(1) = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$

G: $\langle (1,2)(3,4,5,6),\ (3,5)(4,6),\ (2,4,6),\ (1,3,5)(2,4,6) \rangle < \mathfrak{S}_6$
G^0: *G(18,4)* :$(\mathbb{Z}_3)^2 \rtimes_\psi \mathbb{Z}_2$, $\psi(1) = \varphi(2)$
T: $(2,2,3,3)$
L: $(3,5)(4,6),\ (2,6)(3,5),\ (1,3,5),\ (1,5,3)(2,4,6)$
H_1: $\mathbb{Z}_3$
π_1: $\mathbb{Z}_3$

$K^2 = 2$**, Basket** $\{2 \times C_{4,1},\ 3 \times C_{2,1}\}$**.**

1.6. Galois group G(128,1535):

G: $\langle g_1,\ g_2,\ g_3,\ g_4 \mid g_1^{-1}g_4g_1g_4,\ g_4^4,\ (g_2^{-1}g_3^{-1})^2,\ g_2^4,\ (g_3, g_4^{-1}),\ (g_3^{-1}g_2)^2,$
$g_2^{-1}g_4g_2^{-1}g_4^{-1},\ g_1^{-1}g_2^{-1}g_1g_2^{-1},\ g_1^{-1}g_3^{-1}g_1^2g_3g_1^{-1},\ g_3^{-2}g_1g_3^2g_1^{-1},$
$g_4^{-2}g_1g_3g_2^2g_1^{-1}g_3^{-1},\ g_4^{-2}g_3^{-1}g_1g_3g_1^{-1}g_2^2,\ g_4^2g_1^{-2}g_3^{-1}g_2^2g_3^{-1},$
$g_4^{-1}g_1^{-1}g_2g_3g_2^{-1}g_4g_3^{-1}g_1^{-1},\ g_4^{-2}g_1^3g_4^{-2}g_1 \rangle$
G^0: *G(64,73)*: $\langle h_1,\ h_2,\ h_3 \mid h_1^2,\ h_2^2,\ h_3^2,\ (h_1h_3)^4,\ (h_1h_2)^4,$
$(h_2h_3)^4,\ (h_2h_3h_2h_1h_3)^2,\ (h_1h_2h_3h_1h_3)^2,\ (h_2h_1h_3)^4 \rangle$
T: $(2,2,2,4)$
L: $g_1g_3g_4^{-1}g_2^2,\ g_1g_3g_2^{-2}g_3^{-2}g_2^2,\ g_2g_3,\ g_2g_3g_4g_2^{-2}g_4^{-2}g_2^2g_3^{-2}g_2^2$
H_1: $(\mathbb{Z}_2)^3$
π_1: $(\mathbb{Z}_2)^3$

$K^2 = 2$**, Basket** $\{2 \times C_{3,1},\ 2 \times C_{3,2}\}$**.**

1.7. Galois group G(768,1083540):

G: $\langle g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9 \mid g_1^3,\ g_2^2(g_5g_6g_7)^{-1},\ g_3^2(g_5g_6),\ g_4^2(g_5)^{-1},$
$g_5^2,\ g_6^2,\ g_7^2,\ g_8^2,\ g_9^2,\ (g_2,\ g_1)(g_4g_6g_7g_9)^{-1},\ (g_3,\ g_1)(g_3g_7g_9)^{-1},$
$(g_3,\ g_2)g_5^{-1},\ (g_4,\ g_1)(g_8g_9)^{-1},\ (g_4,\ g_2)g_6^{-1},\ (g_4,\ g_3)g_7^{-1},$
$(g_5,\ g_1)(g_6g_7)^{-1},\ (g_5,\ g_2)g_8^{-1},\ (g_5,\ g_3)g_9^{-1},\ (g_6,\ g_1)g_8^{-1},$
$(g_6,\ g_2) = g_8g_9,\ (g_6,\ g_3)g_9^{-1},\ (g_6,\ g_4)g_8^{-1},\ (g_7,\ g_1)g_9^{-1},\ (g_7,\ g_2)g_9^{-1},$
$(g_7,\ g_3)g_8^{-1},\ (g_7,\ g_4)g_9^{-1},\ (g_8,\ g_1)g_9^{-1},\ (g_9,\ g_1) \rangle$
G^0: *G(384,4)*: $\langle h_1,\ h_2 \mid h_1^3,\ h_2^4,\ (h_2^{-1}h_1)^3,\ (h_2^{-1}h_1^{-1})^6,\ (h_2, h_1)^4,$
$h_1^{-1}h_2^{-2}h_1h_2^{-2}h_1^{-1}h_2^{-1}h_1^{-1}h_2h_1^{-1}h_2^{-1},$
$h_2^{-1}h_1h_2h_1h_2^{-1}h_1^{-1}h_2h_1h_2h_1^{-1}h_2^{-1}h_1^{-1}h_2h_1h_2^{-1}h_1^{-1},\ \rangle$
T: $(3,3,4)$
L: $g_1^2g_4g_9,\ g_1g_6g_7g_9,\ g_2g_5g_8$
H_1: $\mathbb{Z}_4$
π_1: $\mathbb{Z}_4$

1.8. Galois group G(768,1083541):

G: $\langle g_1,\ g_2,\ g_3 \mid g_1^3,\ g_3^4,\ g_2^4,\ g_2g_3g_1g_2^{-1}g_1^{-1}g_3,\ g_3^2g_2^2g_3^{-2}g_2^2,$
$g_3g_2^{-1}g_3g_1^{-1}g_2g_3^{-2}g_1,\ g_1^{-1}g_3^{-2}g_1g_2g_3g_2^{-1}g_3^{-1},$

$g_2g_3g_2^{-1}g_1g_2g_1^{-1}g_2^{-1}g_3,\ g_3g_2^2g_3g_1^{-1}g_2g_1g_2,\ (g_3^{-1}g_2^{-1}g_3g_2^{-1})^2,$
$g_2^{-1}g_3^{-1}g_1^{-1}g_3^2g_1g_2g_3,\ (g_3^{-1}g_2)^4,\ g_3g_1g_2^{-2}g_3^{-1}g_2^{-1}g_3^{-1}g_1^{-1}g_2^{-1}g_3,$
$g_3g_2^2g_3^{-1}g_1^{-1}g_3g_2^{-2}g_3^{-1}g_1,\ g_2g_1^{-1}g_2g_1g_3^{-1}g_2g_3g_1g_2g_1^{-1},$
$g_3^{-1}g_2^2g_1^{-1}g_3^{-1}g_1g_3^{-1}g_1^{-1}g_3g_1,\ g_3^{-1}g_2g_3^2g_2g_1^{-1}g_2^2g_1g_3^{-1},$
$g_1^{-1}g_2g_3^{-1}g_2g_3^{-1}g_1g_3^{-2}g_2^2,\ g_3g_1^{-1}g_3g_1g_3^{-1}g_2^{-2}g_3g_2^{-1}g_1g_3g_1^{-1},$
$g_2^{-1}g_1^{-1}g_2g_3^{-1}g_1g_3^{-1}g_1^{-1}g_3g_2^{-2}g_1g_3,$
$g_3^{-1}g_1^{-1}g_2^{-1}g_3^{-1}g_2^{-1}g_1g_3g_2^{-1}g_3^{-1}g_1g_3^{-2}g_2g_3^{-1}g_1^{-1}g_2^{-1}\rangle$

G^0: *G(384,4)*, as above.

T: $(3,3,4)$

L: $g_1^2g_2g_3g_2^{-2}g_3g_2^2g_3g_2^2g_1g_3^{-1}g_2^2g_3g_2^2g_1^{-1},\ g_1g_2^3g_3g_2^2g_1g_3^{-1}g_2^2g_3g_2^2g_1^{-1},$
$g_2g_3^{-1}g_1^{-1}g_3g_1g_2^{-1}g_3^{-2}g_2^{-1}g_3^{-1}g_2^3g_3g_2^2$

H_1: $(\mathbb{Z}_2)^2$

π_1: $(\mathbb{Z}_2)^2$

$K^2 = 3$, Basket $\{C_{8,3},\ C_{8,5}\}$.

1.9. Galois group G(64,42):

G: $\langle(1,2,3,5,8,13,6,10)(4,7,11,14,15,16,9,12),$
$(2,4)(3,6)(5,9)(7,12)(10,11)(13,15)(14,16)\rangle < \mathfrak{S}_{16}$

G^0: *G(32,39)*: $\langle(2,4)(5,7)(6,8)(9,11)(10,12)(13,15),$
$(1,2)(3,5)(4,6)(7,9)(8,10)(11,13)(12,14)(15,16),$
$(1,3)(2,5)(4,7)(6,9)(8,11)(10,13)(12,15)(14,16)\rangle < \mathfrak{S}_{16}$

T: $(2,2,2,8)$

L: $(2,13)(4,15)(5,10)(9,11),$
$(1,7)(2,5)(3,12)(4,15)(6,14)(8,16)(10,13),$
$(2,15)(3,6)(4,13)(5,11)(7,12)(9,10)(14,16),$
$(1,7,3,14,8,16,6,12)(2,15,10,11,13,4,5,9)$

H_1: $\mathbb{Z}_2 \times \mathbb{Z}_4$

π_1: $\mathbb{Z}_2 \times \mathbb{Z}_4$

$K^2 = 4$, Basket $\{4 \times C_{2,1}\}$.

1.10. Galois group G(32,7): $D_{2,8,5} \rtimes_\varphi \mathbb{Z}_2$, $\varphi(1) = \begin{cases} x \mapsto x \\ y \mapsto yxy^4 \end{cases}$

G: $\langle(1,2,3,6,4,5,7,8),\ (2,5)(3,7),\ (2,5)(6,8),\ (1,3,4,7)(2,6,5,8),$
$(1,4)(2,5)(3,7)(6,8)\rangle < \mathfrak{S}_8$

G^0: *G(16,11)*: $D_4 \times \mathbb{Z}_2$

T: $(2,2,2,2,2)$

L: $(2,5)(6,8), (1,7)(2,6)(3,4)(5,8), (1,4)(2,5), (1,4)(2,5),$
$(1,7)(2,8)(3,4)(5,6)$

H_1: $\mathbb{Z}_2 \times \mathbb{Z}_8$

π_1: *G(32,5)*: $(\mathbb{Z}_2)^2 \rtimes_\psi \mathbb{Z}_8$, $\psi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

1.11. Galois group G(32,22): $((\mathbb{Z}_2)^2 \rtimes_\varphi \mathbb{Z}_4) \times \mathbb{Z}_2$, $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

G: $\langle(1,2,5,8)(3,7,10,14)(4,6,11,13)(9,12,15,16),$
$(2,6)(7,12)(8,13)(14,16),$
$(1,3)(2,7)(4,9)(5,10)(6,12)(8,14)(11,15)(13,16),$
$(1,4)(2,6)(3,9)(5,11)(7,12)(8,13)(10,15)(14,16),$
$(1,5)(2,8)(3,10)(4,11)(6,13)(7,14)(9,15)(12,16)\rangle < \mathfrak{S}_{16}$
G^0:*G(16,14)*: $\mathbb{Z}_2^4$
T: $(2,2,2,2,2)$
L: $(1,5)(2,13)(3,10)(4,11)(6,8)(7,16)(9,15)(12,14),$
$(1,3)(2,12)(4,9)(5,10)(6,7)(8,16)(11,15)(13,14),$
$(1,4)(3,9)(5,11)(10,15),$
$(1,10)(2,16)(3,5)(4,15)(6,14)(7,13)(8,12)(9,11),$
$(1,4)(2,6)(3,9)(5,11)(7,12)(8,13)(10,15)(14,16)$
H_1: $(\mathbb{Z}_2)^3 \times \mathbb{Z}_4$
π_1: $\langle p_1, p_2, p_3, p_4 \mid p_1^2, p_3^2, (p_3p_2)^2, (p_1p_2^{-1})^2, p_4p_2^{-1}p_4^{-1}p_2^{-1},$
$p_4p_1p_3p_4^{-1}p_3p_1, (p_1p_4^2)^2, (p_4^{-2}p_3)^2\rangle$

1.12. Galois group G(128,836): Sylow 2-subgroup of a double cover of the Suzuki group $Sz(8)$

G: $\langle(2,4,9,13)(3,7,12,15)(8,10)(11,16),$
$(1,2,5,9)(3,6)(4,10,13,8)(7,11)(12,14)(15,16),$
$(1,3,8,7)(2,6,4,11)(5,12,10,15)(9,14,13,16)\rangle < \mathfrak{S}_{16}$
G^0: *G(64,23)*: $(\mathbb{Z}_2^2 \times \mathbb{Z}_4) \rtimes_\varphi \mathbb{Z}_4$, $\varphi(1) = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$
T: $(4,4,4)$
L: $(1,12,8,15)(2,14,4,16)(3,10,7,5)(6,13,11,9),$
$(1,13,5,4)(2,8,9,10)(3,11)(6,7)(12,16)(14,15),$
$(1,14,8,16)(2,3,13,15)(4,7,9,12)(5,6,10,11)$
H_1: $(\mathbb{Z}_2)^3$
π_1: *G(32,33)*: $(\mathbb{Z}_4^2) \rtimes_\psi \mathbb{Z}_2$, $\psi(1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$

$K^2 = 8$**, Basket** $\emptyset$**.**

1.13. Galois group G(64,92): $(D_{2,8,5} \rtimes_\varphi \mathbb{Z}_2) \times \mathbb{Z}_2$, $\varphi(1) = \begin{cases} x \mapsto x \\ y \mapsto yxy^4 \end{cases}$

G: $\langle(1,2,4,8,5,9,12,16)(3,7,10,15,11,6,13,14),$
$(2,6)(4,12)(7,9)(8,15)(10,13)(14,16),$
$(1,3)(2,7)(4,10)(5,11)(6,9)(8,15)(12,13)(14,16),$
$(1,3)(2,6)(4,10)(5,11)(7,9)(8,14)(12,13)(15,16),$
$(1,4,5,12)(2,8,9,16)(3,10,11,13)(6,14,7,15),$
$(1,5)(2,9)(3,11)(4,12)(6,7)(8,16)(10,13)(14,15)\rangle < \mathfrak{S}_{16}$
G^0: *G(32,46)*: $D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
T: $(2,2,2,2,2)$
L: $(1,5)(2,7)(3,11)(6,9)(8,14)(15,16),$

$(2,7)(4,12)(6,9)(8,14)(10,13)(15,16)$,
$(1,13)(2,8)(3,12)(4,11)(5,10)(6,14)(7,15)(9,16)$,
$(1,4)(2,14)(3,10)(5,12)(6,8)(7,16)(9,15)(11,13)$,
$(1,3)(2,6)(4,10)(5,11)(7,9)(8,14)(12,13)(15,16)$

H_1: $(\mathbb{Z}_2)^3 \times \mathbb{Z}_8$

π_1: $1 \to \Pi_{17} \times \Pi_{17} \to \pi_1 \to G \to 1$

1.14. Galois group G(256,3678):

G: $\langle g_1, g_2, g_3 \mid g_1^4, g_2^4, g_3^4, g_1 g_2 g_3^2 g_1^{-1} g_2^{-1},$
$g_2^{-1} g_1^2 g_3^{-1} g_2^{-1} g_3, g_3 g_1^{-1} g_2^{-1} g_3^{-1} g_1^{-1} g_2, g_1 g_2 g_3 g_2^{-1} g_1 g_3,$
$g_3 g_1^{-1} g_2^{-1} g_1 g_2 g_3, g_2^2 g_3 g_1^{-1} g_3 g_1, g_3 g_1 g_2^{-1} g_3^{-1} g_2^{-1} g_3^{-1} g_1 g_3,$
$g_2^{-1} g_1 g_2 g_1^2 g_3^{-2} g_1, g_1 g_2^2 g_1 g_3^{-1} g_1 g_3^{-1} g_1, g_2^{-2} g_1^{-1} g_3^{-1} g_1 g_3^3,$
$g_3^{-1} g_1 g_2^{-1} g_3^{-2} g_1^{-1} g_3^2 g_2 g_3^{-1} \rangle$

G^0: *G(128,36)*: $\langle h_1, h_2 \mid h_2^4, h_1^4, h_1 h_2^2 h_1^{-2} h_2^{-2} h_1, (h_2^{-1} h_1 h_2 h_1)^2, (h_1, h_2)^2,$
$(h_1^{-1} h_2^{-1} h_1 h_2^{-1})^2, (h_1^{-1} h_2^{-1} h_1^{-2} h_2 h_1^{-1})^2, (h_2^2 h_1^{-1} h_2^2 h_1)^2 \rangle$

T: $(4,4,4)$

L: $g_2 g_3, g_3 g_2^{-1} g_3^{-1} g_2 g_3 g_1^{-1} g_3 g_1 g_3 g_2 g_3^{-2} g_2 g_3^2, g_2^{-1} g_3^2 g_2^{-1} g_3^{-1} g_2 g_3$

H_1: $(\mathbb{Z}_4)^3$

π_1: $1 \to \Pi_9 \times \Pi_9 \to \pi_1 \to G \to 1$

1.15. Galois group G(256,3678):

G: as above

G^0: *G(128,36)*, as above

T: $(4,4,4)$

L: $g_1 g_3^{-2} g_1^{-1} g_3 g_1 g_3 g_2^2, g_3 g_2^{-2} g_3^2 g_2^{-1} g_3^{-1} g_2 g_3, g_1 g_3^{-1} g_2^{-2} g_3^2 g_1^{-1} g_3 g_1 g_3 g_2 g_3^{-2} g_2 g_3^2$

H_1:$(\mathbb{Z}_2)^4 \times \mathbb{Z}_4$

π_1: $1 \to \Pi_9 \times \Pi_9 \to \pi_1 \to G \to 1$

1.16. Galois group G(256,3678):

G: as above

G^0: *G(128,36)*, as above

T: $(4,4,4)$

L: $g_1 g_3 g_2^{-2} g_3^2 g_2^{-1} g_3^{-2} g_2 g_3^2, g_1 g_2 g_3 g_1^{-1} g_3 g_1 g_3 g_2^2, g_2 g_3^{-2} g_1^{-1} g_3 g_1 g_3 g_2^2$

H_1: $(\mathbb{Z}_2)^2 \times (\mathbb{Z}_4)^2$

π_1: $1 \to \Pi_9 \times \Pi_9 \to \pi_1 \to G \to 1$

1.17. Galois group G(256,3679):

G: $\langle g_1, g_2, g_3 \mid g_3^4, g_1^4, g_2^4, g_2 g_3^2 g_1^{-1} g_2^{-1} g_1, g_3^{-1} g_2^{-1} g_3^{-1} g_1 g_2^{-1} g_1,$
$g_3^{-1} g_2 g_3 g_2 g_1^2, g_2^{-1} g_1 g_2^{-1} g_3^{-1} g_1^{-1} g_3, g_1^2 g_2^{-1} g_3^{-1} g_2^{-1} g_3, g_2^{-1} g_3 g_2 g_1 g_3 g_1,$
$g_1^{-1} g_2^{-1} g_1^2 g_3 g_1^{-1} g_3^{-1} g_2^{-1}, g_3^{-1} g_2 g_3 g_2^{-1} g_1^{-2} g_2^{-2}, (g_3^{-1} g_2)^4,$
$g_2^{-1} g_1 g_2^{-1} g_1 g_3^{-1} g_1 g_3 g_1, \rangle$

G^0: *G(128,36)*, as above

T: $(4,4,4)$

L: $g_2 g_3, g_3 g_2^{-1} g_3^{-1} g_2 g_3 g_1 g_3^2 g_1^{-1} g_3^{-2} g_2^{-1} g_3^{-2} g_2 g_3^{-2}, g_2^{-1} g_3^2 g_2^{-1} g_3^{-1} g_2 g_3$

H_1: $(\mathbb{Z}_2)^2 \times (\mathbb{Z}_4)^2$

π_1: $1 \to \Pi_9 \times \Pi_9 \to \pi_1 \to G \to 1$

$K^2 = 2$, Basket $\{C_{2,1},\, 2 \times D_{2,1}\}$.

2.1. Galois group G(4,1): $\mathbb{Z}_4$
G: $\mathbb{Z}_4$
G^0: *G(2,1)*: $\mathbb{Z}_2$
T: (2,2)
L: 2, 0, 2, 2
H_1: $\mathbb{Z}^2$

2.2. Galois group G(16,8): $D_{2,8,3}$
G: $D_{2,8,3}$
G^0: *G(8,3)*: D_4
T: (2)
L: x, xy^2, y^4
H_1: $\mathbb{Z}^2$

2.3. Galois group G(16,9): BD_4
G: BD_4
G^0:*G(8,4)*: Q_8
T: (2)
L: y^6, yx, y^4
H_1: $\mathbb{Z}^2$

$K^2 = 4$, Basket $\{4 \times C_{2,1}\}$.

2.4. Galois group G(8,1): $\mathbb{Z}_8$
G: $\mathbb{Z}_8$
G^0: *G(4,1)*: $\mathbb{Z}_4$
T: (2,2)
L: 2,0,4,4
H_1: $\mathbb{Z}_2 \times \mathbb{Z}^2$

2.5. Galois group G(8,2): $\mathbb{Z}_2 \times \mathbb{Z}_4$
G: $\mathbb{Z}_2 \times \mathbb{Z}_4$
G^0: *G(4,2)*: $\mathbb{Z}_2 \times \mathbb{Z}_2$
T: (2,2)
L: (1,0),(1,2),(1,0),(1,0),
H_1: $\mathbb{Z}_2 \times \mathbb{Z}^2$

2.6. Galois group G(32,29):
G: $\langle x, y, z \mid z^2,\, y^4,\, x^4,\, x^{-1}y^2x^{-1},\, x^{-1}y^{-1}xy^{-1},\, xy^{-1}xy,\, y^{-1}zyz,\, zy^{-1}x^2zy^{-1},\, (zxzx^{-1})^2\rangle$
G^0: *G(16,3)*: $(\mathbb{Z}_2)^2 \rtimes_\varphi \mathbb{Z}_4$: $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
T: (2)
L: z^2xzx^{-1}, x, $zxzx^{-1}$
H_1: $\mathbb{Z}_2^2 \times \mathbb{Z}^2$

2.7. Galois group G(32,13): $D_{4,8,3}$
G: $D_{4,8,3}$

G^0: *G(16,4)*: $D_{4,4,3}$
T: (2)
L: y^6, x, y^4
H_1: $\mathbb{Z}_2 \times \mathbb{Z}^2$

2.8. Galois group G(32,14): $D_{4,8,-1}$
G: $D_{4,8,-1}$
G^0: *G(16,4)*: $D_{4,4,3}$
T: (2)
L: y^2, x, y^4
H_1: $\mathbb{Z}_2 \times \mathbb{Z}^2$

2.9. Galois group G(32,32):
G: $\langle x, y, z \mid y^4,\, x^4,\, x^2z^2,\, x^{-1}yx^{-1}y^{-1},\, (y, z^{-1}),\, y^{-1}xzx^{-1}z^{-1}y^{-1}\rangle$
G^0: *G(16,4)*: $D_{4,4,3}$
T: (2)
L: $yz^{-1}y^{-2}$, xz, $z^{-2}y^{-2}$
H_1: $\mathbb{Z}_2^2 \times \mathbb{Z}^2$

2.10. Galois group G(32,35):
G: $\langle x, y, z \mid x^2y^2,\, x^{-1}y^{-1}x^{-1}y,\, z^4,\, x^4,\, x^{-1}zxz,\, (y, z^{-1})\rangle$
G^0: *G(16,4)*: $D_{4,4,3}$
T: (2)
L: xy^{-2}, z^{-1}, z^{-2}
H_1: $\mathbb{Z}_2^2 \times \mathbb{Z}^2$

2.11. Galois group G(32,15):
G: $\langle x, y \mid x^{-1}y^3xy^{-1},\, x^{-1}yx^{-1}yx^{-2},\, xy^{-1}x^{-2}yx,\, xy^{-1}x^{-1}y^{-1}x^{-1}y^{-1}xy^{-1},\rangle$
G^0: *G(16,8)*: $D_{2,8,5}$
T: (2)
L: y^{-2}, x, y^{-4}
H_1: $\mathbb{Z}_2 \times \mathbb{Z}^2$

$K^2 = 5$**, Basket** $\{C_{3,1},\, C_{3,2}\}$**.**

2.12. Galois group G(24,4): BD_6
G: BD_6
G^0: *G(12,1)*: BD_3
T: (3)
L: $y^8, xy^3, y^8,$
H_1: $\mathbb{Z}_2 \times \mathbb{Z}^2$

2.13. Galois group G(24,5): $D_{2,12,5}$
G: $D_{2,12,5}$
G^0:*G(12,4)*: D_6
T: (3)
L: y^4x, y^4, y^2x
H_1: $\mathbb{Z}_2^2 \times \mathbb{Z}^2$

$K^2 = 6$**, Basket** $\{2 \times C_{2,1}\}$**.**

2.14. Galois group G(48,30): $A_4 \rtimes_\varphi \mathbb{Z}_4$,

G: $A_4 \rtimes_\varphi \mathbb{Z}_4$, $\varphi_1(1) = \begin{cases} (1,2)(3,4) & \mapsto & (1,2)(3,4) \\ (1,2,3) & \mapsto & (2,3,4) \end{cases}$
G^0: *G(24,13)*: $A_4 \times \mathbb{Z}_2$
T: (2)
L: $((1,3)(2,4);0)$, $((2,4,3);0)$, $((1,2,3);2)$
H_1: $\mathbb{Z}_2^2 \times \mathbb{Z}^2$

2.15. Galois group G(48,31): $A_4 \times \mathbb{Z}_4$
G: $A_4 \times \mathbb{Z}_4$
G^0: *G(24,13)*: $A_4 \times \mathbb{Z}_2$
T: (2)
L: $((1,2)(3,4);0)$, $((1,3,4);0)$, $((1,4)(2,3);2)$
H_1: $\mathbb{Z}_2^2 \times \mathbb{Z}^2$

$K^2 = 6$**, Basket** $\{C_{5,3}\}$**.**

2.16. Galois group G(20,3): $\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$
G: $\mathbb{Z}_5 \rtimes_\varphi \mathbb{Z}_4$, $\varphi(1) = (1 \mapsto 3)$
G^0: *G(10,1)*: D_5
T: (5)
L: (3,0), (1,0), (2,2)
H_1: $\mathbb{Z}_2^2 \times \mathbb{Z}^2$

$K^2 = 8$**, Basket** $\emptyset$**.**

2.17. Galois group G(16,6): $D_{2,8,5}$
G: $D_{2,8,5}$
G^0: *G(8,2)*: $\mathbb{Z}_2 \times \mathbb{Z}_4$
T: (2,2)
L: y^2, xy^4, xy^4, xy^4
H_1: $\mathbb{Z}_4 \times \mathbb{Z}^2$

2.18. Galois group G(16,8): $D_{2,8,3}$
G: $D_{2,8,3}$
G^0: *G(8,3)*: D_4
T: (2,2)
L: $Id(G), xy^4, xy^2, xy^2$
H_1: $\mathbb{Z}_4 \times \mathbb{Z}^2$

2.19. Galois group G(16,3): $(\mathbb{Z}_2)^2 \rtimes_\varphi \mathbb{Z}_4$: $\varphi(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

G: $\langle (1,2,4,6)(3,5,7,8), (2,5)(6,8), (1,3)(2,5)(4,7)(6,8),$
$(1,4)(2,6)(3,7)(5,8) \rangle < \mathfrak{S}_8$
G^0: *G(8,5)*: $\mathbb{Z}_2^3$
T: (2,2)

L: (14)(28)(37)(56),(25)(68), (17)(26)(34)(58)
H_1: $\mathbb{Z}_2^3 \times \mathbb{Z}^2$

3. $p_g = q = 2$

$K^2 = 8$, Basket $\emptyset$.

3.1. Galois group G(4,1): $\mathbb{Z}_4$
G: $\mathbb{Z}_4$
G^0: *G(2,1)*: $\mathbb{Z}_2$
T: –
L: $1, 1, 0, 1$
H_1: $\mathbb{Z}_2 \times \mathbb{Z}^4$